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TWO PROPOSITIONS RELATED TO PUBLIC GOODS

Robert H. Strotz

THIS note, pertinent to some recent literature on public goods, presents two propositions which place the question of the optimal expenditure levels for public goods in a somewhat different perspective. By (pure) public goods or, synonymously, (pure) collective consumption goods, we mean those consumer goods having the property that, once produced, their enjoyment by each and every individual does not reduce their availability for the enjoyment of others. Public defense and public health measures may suggest cases in point.

I

It was natural and proper for Professor Samuelson in his recent elucidation of the problem of determining the optimal amounts of social goods for society to produce to formulate and treat this problem in the context of allocation theory. Given a social welfare function, i.e., some criterion for choosing among all the Pareto-optimal welfare positions, what outputs of various social goods are indicated? Our purpose here is to cast this problem of welfare economics in a somewhat different way and to show that, within limits, decisions about the production of social goods should be regarded as decisions about the distribution of income which are independent of questions of economic inefficiency. We adopt the viewpoint of the New (Pareto) Welfare Economics and Professor Samuelson's mode of analysis.

Let

\[ u^i = u^i(x_{i1}, ..., x_{iN}, S_1, ..., S_K), \]

be the ordinal utility functions for the \( I \) individuals in the economy where the \( x_{it} \)'s represent private goods consumed by individuals. Some \( x_{it} \)'s may be defined as the negative values of certain factor services (e.g., labor services) which enter the utility functions negatively. \( S_1, ..., S_K \) stand for amounts of various public goods produced. The marginal utilities of these public goods are all assumed to be positive, i.e., \( \partial u^i / \partial S_k = u_{ik} > 0, \ i = 1, ..., I; \ k = 1, ..., K. \) (If some individuals were not desirous of certain public goods, some of these partial derivatives would be zero. This would complicate the mathematical treatment somewhat without affecting the essence of our argument.)

There is, moreover, a social transformation function

\[ T(\sum_{i=1}^{I} x_{i1}, ..., \sum_{i=1}^{I} x_{iN}; S_1, ..., S_K) = 0 \]

(2)

describing the production possibilities open to the economy.

We assume, in addition, that the private goods sector of the economy is organized so as to satisfy the usual optimal exchange conditions, each individual maximizing his utility subject to fixed common prices (and wage rates) and to any fixed income arising at least in part from whatever social dividend he may receive. Therefore,

\[ u^i = p_n u_{ni}, \quad i = 1, ..., I; \quad n = 1, ..., N-1; \]

(3)

and

\[ \sum_{n=1}^{N} x_{ni} = s_i, \quad i = 1, ..., I; \]

(4)

justified by the feasibility consideration that, even if these are not 100 per cent efficient in avoiding avoidable deadweight loss, they may be better than the attainable imperfect tax alternatives. (“Diagrammatic Exposition . . . ,” 356.)

My purpose is to look into this point somewhat systematically.


2 To avoid misunderstanding, let me say that I do not suppose that my argument would come to Samuelson as news. He already touched upon the issues considered here when he wrote: “I am anxious to clear myself from Dr. Margolis’ understandable suspicion that I am the type of liberal who would insist that all redistributions take place through tax policies and transfer expenditures; much public expenditure on education, hospitals, and so on, can be...” (Pareto) Welfare Economics and Professor Samuelson’s mode of analysis.

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where \( u^i_n \) is \( \partial u^i / \partial x^i_n \), \( p_n \) is the price of commodity \( n \) (\( p_n = 1 \), the \( N \)th commodity serving as numéraire); and \( s^i \) is that component of individual \( i \)'s income deriving from the ownership of fixed-income assets (whose productive services do not appear among the \( x^i_n \)'s and whose supply is perfectly price-inelastic) and his social dividend (transfer receipts).

The remaining requirement is that
\[
\sum s^i = H, \quad H \text{ a constant.} \tag{5}
\]
This conserves "purchasing power" in the economy by requiring the algebraic sum of transfer payments to be zero.\(^4\)

To find the Pareto optima we extremize \( \sum w^iu^i \), where the \( w^i \) are "weights," \( w^i > 0, i = 1, \ldots, I \), for all possible values of the \( w^i \)'s, provided only that these weights are normalized, e.g., by \( w^i = 1 \).\(^5\)

We therefore form the auxiliary Lagrangean equation
\[
\sum w^iu^i \left( \ldots x^i_n; \ldots S^\prime_k \ldots \right) + \sum T \left( \ldots \Xi x^i_n \ldots \right)
+ \sum \lambda \left[ a \left( u^i_n - p_n x^i_n \right) \right] + \sum \mu \left[ T - \sum s^i \right] + \mu \left[ \sum s^i - H \right], \tag{6}
\]
where \( \kappa \), the \( \lambda_n \)'s, the \( \lambda^i \)'s, and \( \mu \) are undetermined multipliers. This is to be extremized with respect to the \( x^i_n \)'s, the \( S^i_k \)'s, the \( p_n \)'s, and the \( s^i \)'s for a given vector of \( w^i \)'s (\( w^i = I \)). We thereby obtain the conditions
\[
\sum w^iu^i_k + \kappa T = 0, \quad k = 1, \ldots, K, \tag{8}
\]
\[
- \sum \lambda u^i_n + \sum \lambda x^i_n = 0, \quad n = 1, \ldots, N - 1, \tag{9}
\]
and
\[
- \lambda^i + \mu = 0, \quad i = 1, \ldots, I. \tag{10}
\]
Equations (1)-(5) and (7)-(10) are
\[
I + I - I + I + I + I + I + K + N - I + I =
2I + K + 2I + N + I
\]
and
\[
I + I + I + I + I + I + I + I + I + I = 2I + K + 2I + N + I
\]
in number. Taking the \( I - 1 \) \( w^i \)'s, \( i = 1, \ldots, I - 1 \), as given, we then have \( I + I + K + N - I + I + I + I = 2I + K + 2I + N + I \) unknowns.

The usual loose criterion for determinacy (finding equations and unknowns equal in number) is therefore satisfied.

### II

Given then a set of weights, the \( w^i \)'s, which serve in place of a social welfare function, there will be a determinate solution for the output of public goods, say, \( S^0_1, \ldots, S^0_K \). Any other decision about the amounts of these goods may be expected not to lead to the same optimum. But will a different and arbitrary decision, say, \( S^*_i, \ldots, S^*_K \), lead simply to another Pareto optimum or to a Pareto-inferior position?

If the answer is the latter, then in deciding upon the amounts of social goods to produce, the problem of inefficiency in production is encountered. If the answer is the former, then a decision as to the \( S^i_k \)'s is a choice among Pareto optima and we shall say the problem is a strictly distributional one.

Suppose we assume next that the number of public goods to be considered is less than the number of individuals, i.e., that \( K < I \). Then treat the \((I - 1)w^i\)'s as variables (remembering that \( w^i = I \)). Equations (1)-(5) and (7)-(10) may then be solved for those \( w^i \)'s. Moreover, since \( K \) may be less than \( I - 1 \), the solution need not be unique. If one possible solution has the property that
\[
w^i > 0, \quad i = 1, \ldots, I - 1,
\]
the arbitrary choice of \( S^i_1, \ldots, S^i_K \) will have led to a Pareto optimum. The optimum will require, however, the appropriate selection of \( s^i \)'s. This is to say that provided \( K < I \) and \( w^i > 0, i = 1, \ldots, I - 1 \), an arbitrary decision as to the amounts of public goods to be produced will be optimal provided that transfer payments are
appropriately chosen. These transfer payments can, of course, be regarded as the individual assessments levied to "pay" for the production of the social goods. Indeed, if the solution for the \( w^i \)'s is not unique subject to the condition that they all be positive, then the solution for the \( s^i \)'s will not be unique either. We may count up to \( I - 1 - K \) "degrees of freedom" for the \( s^i \)'s, in the sense that we may be able to choose values for that many \( s^i \)'s quite arbitrarily (within what may be quite broad limits).

As a practical matter transfer payments cannot be placed on an individual basis. This would require the feasibility of as many different flat taxes and subsidies as there are individuals in the economy. So many "degrees of freedom" are simply not available to the State. If account is taken of this, we may wish to modify the model by requiring that

\[
\begin{align*}
s^1 &= s^2 = \ldots = s^I, \\
s^1 + 1 &= s^1 + 2 = \ldots = s^2, \\
\vdots &\quad \vdots \\
s^J + 1 &= s^J + 2 = \ldots = s^I,
\end{align*}
\]

there being only \( J + 1 \) transfer payments or receipts to be chosen, the same payments or receipts applying uniformly within \( J + 1 \) groups of individuals. Under these circumstances equation set (10) consists of only \( J + 1 \) equations.

If \( K + J + 1 \leq I - 1 \), as would seem realistic in a political context, the \( J + 1 \) transfer payments and receipts and the \( K \) amounts of public goods can be chosen arbitrarily and the system solved for \((K + J + 1)w^i\)'s (the remainder being chosen arbitrarily). If all \( w^i \)'s are found to be positive, a Pareto optimum results. This is to say that provided \( K + J + 1 \leq I - 1 \) and \( w^i > 0, i = 1, \ldots, I - 1 \), an arbitrary decision as to the amounts of public goods to be produced will be optimal regardless of what transfer payments are chosen.

If we assume that the \( w^i \)'s can be regarded as continuous functions of the \( S_k \)'s and the \( s^i \)'s, then about any optimal position there will be a range of variation possible in the \( S_k \)'s and \( s^i \)'s so that the resulting welfare position remains optimal. More significantly, we can say that for given \( s^i \)'s there will be a range of possible variations in the \( S_k \)'s for which welfare will be optimal. That a set of \( S_k \)'s be chosen from this region is a question of allocative efficiency, but the particular choice from within this region is a distributional question alone. Indeed, put somewhat loosely, it would appear that if \( K + J + 1 \) is small relative to \( I - 1 \), the danger of an inefficient allocation of resources resulting from any reasonable decision about the production of public goods would also be small.

### III

Now, there is nothing peculiar to the nature of public goods that is responsible for these results. If there is some latitude in picking the \( K S_k \)'s, there is also some latitude in picking any \( K \) other variables that we can get our hands on. For example, we might consider setting any \( K \) prices arbitrarily. Replace \( S_1, \ldots, S_K \) wherever they appear in section II with the prices of the first \( K \) private goods, \( p_1, \ldots, p_K \) (assuming \( K < N \)), and the same argument applies.

Given these \( K \) prices we need "only" try to find transfer payments which make them optimal. Such payments may exist for some range in which these prices have been set. And if the number of different transfer payments possible, \( J + 1 \), is such that \( K + J + 1 \leq I - 1 \), public regulation of the \( K \) prices may be used efficiently to achieve a desired income distribution on a feasibility frontier.\(^6\)

But there is something peculiar to the nature of public goods that makes these results especially interesting. It is that, whereas the output decisions regarding private goods may be left to a free market with prices and amounts sold to various individuals determined by competitive market forces, this is not possible for public goods. Nor is any other very usable solution available. This is precisely the point of Samuelson's original article: "the failure of market cattalactics." What we claim to have shown is that with \( K + J + 1 \leq I - 1 \), this difficult problem can be largely avoided, precisely by seizing upon public goods as supplementary devices for controlling the distribution of income.
